Computationally Efficient Implementation of OFDM Peak-to-Average Power Reduction Technique*

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ABSTRACT

The peak-to-average power of an orthogonal frequency division multiplexed (OFDM) signal can be reduced without increasing the out-of-band power by clipping the amplitude of the oversampled baseband time domain signal and filtering using a frequency domain filter based on discrete Fourier transforms. Clipping the amplitude however requires a division operation and this is undesirable in digital signal processing (DSP) implementations. In this paper it is shown that a computationally efficient amplitude reduction technique based on a look-up table can be used with little degradation in performance.

I. INTRODUCTION

One of the main disadvantages of Orthogonal Frequency Division Multiplexing (OFDM) is its high peak-to-average power ratio (PAPR). OFDM transmitters therefore require very linear output amplifiers with wide dynamic range. These are expensive and inefficient. Any amplifier non-linearity causes intermodulation products resulting in unwanted out-of-band power. Although the PAPR is very large for OFDM, high magnitude peaks occur relatively rarely and most of the transmitted power is concentrated in signals of low amplitude.

The simplest approach to reducing the PAPR of OFDM signals is to clip the high amplitude peaks. A variety of clipping techniques have been described in the literature [1-2]. Some clip the outputs of the inverse fast Fourier transform (IFFT) without interpolation. However the signal must be interpolated before analog-to-digital conversion, and this will cause peak regrowth [3].

To avoid the problem of peak regrowth, the signal can be clipped after interpolation. However this causes very significant out-of-band power. Some papers have described clipping of the interpolated signal followed by filtering [1]. The filters used have been complicated. Filtering also causes peak regrowth, although this is less than for the case of clipping before interpolation.

A recent paper [4] described a new clip-and-filter solution that reduces the PAPR with no increase in out-of-band power. Fig. 1 shows the block diagram of the system. Vector $A_i = a_{0,i} \cdots a_{N-1,i}$, which represents the data in each symbol, is padded with $N(I_1 - 1)$ zero values and input to an NI_1 point inverse DFT. This results in $B_i = b_{0,i} \cdots b_{N(I_1+1)-1,i}$ a vector of the discrete time domain symbol oversampled by a factor of I_1. The discrete time domain signal is then clipped to give the vector $P_i = p_{0,i} \cdots p_{N(I_1+1)-1,i}$. In [4] amplitude clipping was used. At every point where the complex time domain signal exceeded the clipping level, α, the amplitude was reduced to the clipping level while the phase of the complex signal was unchanged.

In this paper the concept of clipping ratio, *CR*, is used. This is normalized to the mean power of the unclipped signal.

$$CR = 10\log_{10}\left(\frac{\alpha^2}{E\left[\left|b_{k,i}\right|^2\right]}\right). \quad (1)$$

Because this is a non-linear process, intermodulation occurs and results in out-of-band power. The clipped signal is then filtered to reduce out-of-band power. The filter consists of two FFT operations. The forward FFT transforms the clipped signal back into the discrete frequency domain. The in-band discrete frequency components of the clipped signal $c_{0,i} \cdots c_{N/2-1,i}, c_{NI_1-N/2+1,i} \cdots c_{NI_1-1,i}$ are passed unchanged to the inputs of the second IFFT while the out-of-band components, $c_{N/2+1,i} \cdots c_{NI_1-N/2,i}$ are nulled. In applications where the band-edge subcarriers are not used, these subcarriers can also be nulled by the filter [5]. The outputs of the second IFFT, $Q_i = q_{0,i} \cdots q_{N(I_2+1)-1,i}$ are the samples of the clipped and filtered baseband signal with an oversampling factor of I_2.

It was shown in [4] that the technique is effective in reducing the PAPR and causes no increase in out-of-band power. The filtering operation causes some peak regrowth. In [6] it was shown that repeated clipping and filtering operations could be used to reduce the overall peak regrowth.

A disadvantage of the technique is that amplitude limiting of the complex baseband signal requires a division operation and this is undesirable in a digital signal processing (DSP) implementation. In this paper an amplitude limiting technique based on a look up table and multiplication is used.

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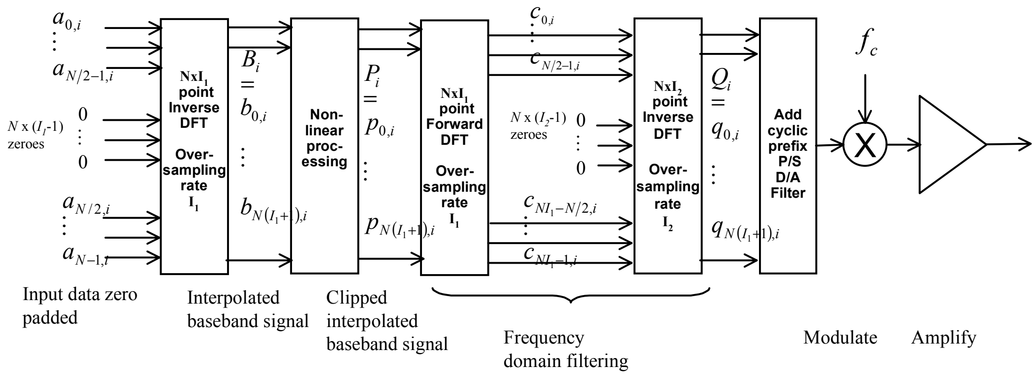


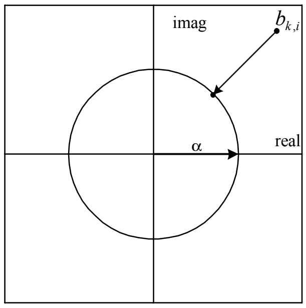
Fig. 1. Block diagram of peak reduction technique

II. PAPR REDUCTION ALGORITHMS

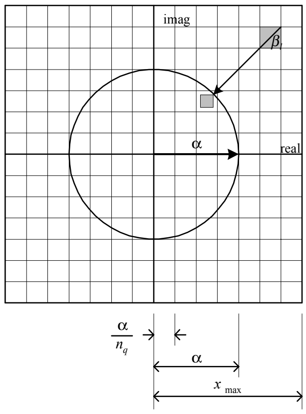
Fig. 2(a) shows simple amplitude limitation. The outputs $b_{k,i}$ of the oversized IFFT are complex values. Points in the complex plane outside a circle of radius α are points that exceed the clipping level. If $|b_{k,i}| > \alpha$ then amplitude limiting reduces the amplitude of the sample to α while the angle of the sample is unchanged. Points within the circle are unchanged by the amplitude limitation. To calculate the values that points outside the circle should be mapped to, the real and imaginary components must be multiplied by $\alpha / |b_{k,i}|$. This involves a division process and this is undesirable in a DSP implementation.

Fig. 2(b) shows a slightly different amplitude reduction technique that can be implemented using a look-up table and a multiplication. The complex plane is divided into squares. A multiplying factor, β_l, is associated with each square. The amplitude limiting process involves identifying within which square each $b_{k,i}$ falls and then calculating $\beta_l b_{k,i}$. Some squares are completely within the circle, for these $\beta_l = 1$ and points within these squares are unchanged by the process. For squares which lie partly or wholly outside the circle β_l is calculated so that it moves the point in the square with the largest magnitude radially onto the circumference of the circle. This has the effect of mapping all of the values within the square onto a smaller square that is completely within the circle.

To implement the technique, a look-up table that contains all of the values of β_l is required. The most significant bits of the real and imaginary components of each $b_{k,i}$ are used to calculate the index into the look-up table as these indicate within which square the point falls. As the value of β_l depends only on the magnitudes of the real and imaginary components and not on whether they are positive or negative, some reduction in the size of the look-up table is possible. A multiplication occurs whether or not the point falls within the circle, so no decision operation is required.



(a)



(b)

Fig. 2. Amplitude limitation techniques considered as mappings on complex plane. (a) simple amplitude clipping, (b) new technique based on look-up table.

The performance of the system depends on the size of the squares. If large squares are used, then only a small look-up table is required, but the process will cause greater signal distortion than necessary as some signal samples are mapped onto points well within the circle rather than on or near the circumference. In the following simulations the size of the squares is defined in relation to α. n_q was defined as the number of quantization steps in α. Thus the squares had sides of length α / n_q.

The distributions of the real and the imaginary components of an OFDM signal are approximately Gaussian, so signals with very large real and imaginary components may occur, although with relatively low probability. In these simulations the look-up table held values only for real and imaginary components with amplitude less than x_{max}. Any $b_{k,i}$ with a real component greater than x_{max} was multiplied by the β_l associated with the square corresponding to a real component of x_{max}, and similarly for the imaginary component. If the symmetry of the mapping is exploited, the number of entries in the look-up table is $\left((x_{max} n_q)/\alpha\right)^2$.

III. PAPR REDUCTION

The most basic measure of the performance of a PAPR reduction scheme is its effectiveness in reducing the PAPR. Fig. 3 shows the complementary cumulative distribution function for clipped (but not filtered) time domain vectors B_i for $CR = 7\text{dB}$ and $x_{max} = 4\alpha$. The results shown are for $N = 64$ and 4QAM modulation but any $N \geq 64$ and any other subcarrier modulation scheme would give similar results. The simulation results are for 1000 symbols. Results are shown for n_q ranging from 1 to 32. The power is calculated relative to the power of the signal after clipping. For $n_q = 32$ the results are almost identical to those for amplitude clipping. There are a negligible number of samples above CR.

For $n_q = 1$ a significant proportion of the signal samples after clipping exceed 7dB. In this case the amplitude of each sample is significantly reduced and so the average power of the clipped signal is significantly less than the average power before clipping.

Fig. 4 shows the complementary cumulative distribution for the clipped and filtered signals for the same cases. It indicates that as long as $n_q \geq 2$ the technique is effective in reducing PAPR. For $n_q \geq 2$, the distribution after filtering is better for fewer quantization steps. This is because the clipping algorithm is reducing many samples well below α and so peak regrowth due to filtering results in a smaller proportion of samples with amplitudes exceeding α. However as will be shown in the next section this is at the cost of greater overall signal distortion.

Further simulations (results not shown) indicate that increasing x_{max} from α to 1.5α improves the distribution but further increase in x_{max} gives no significant further improvement.

Using coarser quantization gives greater reduction in the dynamic range of the signal. But it will be shown in the next section that this is at the cost of increased signal degradation.

IV. IN-BAND DISTORTION

The clip-and-filter technique causes no increase in out-of-band power so the limit to PAPR reduction in a practical system is the in-band distortion, in the form of clipping noise, caused by clipping.

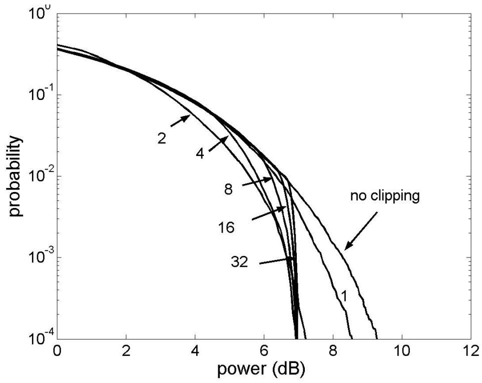


Fig. 3. Complementary cumulative distribution for clipped signal (before filtering) for varying n_a

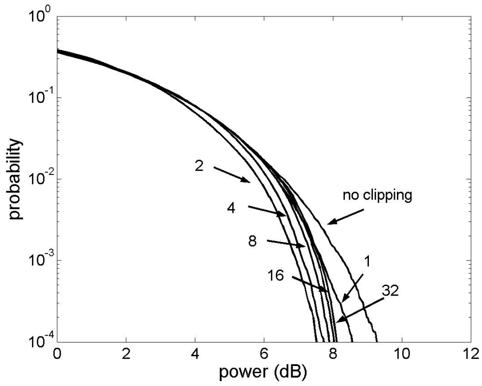


Fig. 4. Complementary cumulative distribution for clipped and filtered signal for varying n_q

A number of authors have analyzed the effects of non-linearities on OFDM signals [7]. The real and imaginary components of an OFDM signal have Gaussian distributions, thus by extending Bussgang's theory to the complex case it is possible to show that, subject to certain conditions [7],

$$y(t) = Kx(t) + d(t), \tag{2}$$

where $x(t)$ is the input to the non-linearity, K is a constant and $y(t)$ is the output. $d(t)$ is a zero mean random noise process which is uncorrelated with $x(t)$. This does not apply precisely in this case as the clipping algorithm cannot be described simply as a nonlinearity; the clipping is a function of both the real and imaginary components of the signal samples. However the effect of clipping is still to cause an overall reduction in signal power and to cause a noise-like distortion. As n_q increases the clipping is more closely approximated by a non-linearity.

In an OFDM system, the signal $y(t)$ is the transmitted time domain signal. In the absence of distortion and noise in the channel, samples of sections of $y(t)$ are input to the receiver FFT. So the vector output from the FFT for the i-th received symbol can be described by

$$Z_i = KA_i + N_{C_i}, \tag{3}$$

where $A_i = a_{0,i} \cdots a_{N-1,i}$. N_{C_i} is the FFT of the vector of samples of $d(t)$. In this case the signal to clipping noise ratio (SCNR) for the k-th subcarrier is given by,

$$SCNR_k = \frac{K^2 E\left[\left(a_{k,i}\right)^2\right]}{E\left[\left(n_{c_{k,i}}\right)^2\right]}, \qquad (4)$$

where $n_{c_{k,i}}$ is the k-th element of the clipping noise vector N_{C_i}.

In general $SCNR_k$ depends on the subcarrier index [7]. In the simulations an average value of the SCNR was calculated. *K* was estimated by measuring the power of the entire sequence after clipping. This value of *K* was then used to calculate the sequence of N_{C_i}. Fig. 5 shows the SCNR for varying n_q and *CR*, with $x_{\max} = 4\alpha$. Even for severe clipping and very coarse quantization, SCNR is quite large. This is because the main effect of clipping is to reduce the overall signal amplitude rather than increase the variance of $n_{c_{k,i}}$. As *CR* increases SCNR also increases because fewer clipping events occur. SCNR increases as n_q increases because finer quantization means that the signal distortion is less.

Fig. 6 shows the dependence of SCNR on $x_{\max}$ for $n_q = 64$. The SCNR increases as $x_{\max}$ decreases. This is because for small $x_{\max}$, the distortion is less because even after mapping, many samples will remain outside the circle.

The distribution of the real (or imaginary) component of $d(t)$ is in general very non-Gaussian, with a large peak at zero corresponding to input signal values which are below the clipping level, and are therefore not clipped. Despite this, because of the action of the FFT, $n_{c_{k,i}}$ depends on all of the clip events within the symbol and may be Gaussian if the central limit theorem applies. This is the case if *CR* is such that there are a significant number of clips per symbol. This will occur at lower *CR* as *N* increases.

The clipping noise is added at the transmitter rather than the receiver. In fading channels this means that in general the clipping noise will cause less degradation in bit error rate than noise added in the channel because the clipping noise fades along with the signal [5].

V. CONCLUSIONS

In this paper it has been shown that a PAPR reduction scheme based on clipping and frequency domain filtering can be implemented using a look-up table and multiplication. This technique is well suited to DSP implementation. The most significant bits of the real and imaginary components of the time domain OFDM signal are used to calculate the index into the look-up table. Good performance can be achieved with $x_{\max} = 1.5\alpha$ and $n_q = 8$. This would require a look-up table with only 144 entries.

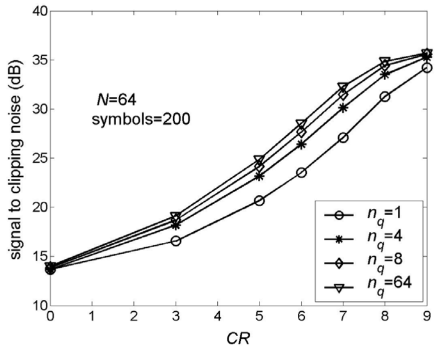


Fig. 5. Signal to clipping noise ratio as a function *CR* for varying n_q

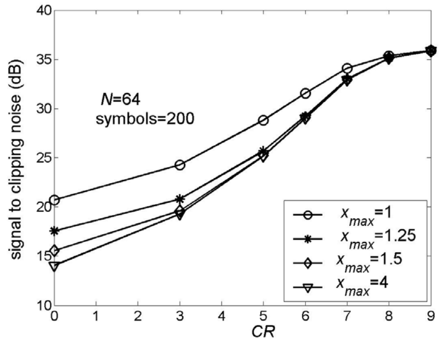


Fig. 6. Signal to clipping noise ratio as a function *CR* for varying x_{max}

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