Blind Estimation of Time and Frequency Offset for PCC-OFDM

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Abstract

This paper presents blind estimation methods for timing and frequency offset in polynomial cancellation coded orthogonal frequency division multiplexing (PCC-OFDM) systems. The estimations are carried out in the frequency domain. The correct timing point is calculated by maximizing a timing metric and the frequency offset estimate is obtained by using pairs of demodulated subcarriers at the Fast Fourier Transform (FFT) output. No training symbols, pilot tones or cyclic prefix extensions are required. The methods can also be used in OFDM, however a training symbol is required.

1. Introduction

Polynomial cancellation coded OFDM (PCC-OFDM) is a data transmission scheme in which the data to be transmitted is mapped onto weighted groups of subcarriers rather than individual subcarriers [1][2]. PCC-OFDM overcomes many disadvantages of OFDM. For example, PCC-OFDM is robust to frequency offset and Doppler spread, has much steeper power spectrum, and therefore much lower out-of-band power than normal OFDM [3]. One important feature of PCC-OFDM is that no cyclic prefix is required [4].

Figure 1 shows the structure of a PCC-OFDM communication system, where $d_{0,i} \cdots d_{n-1,i}$ are n data values in the ith data block to be transmitted. They are mapped onto N subcarriers $a_{0,i}, \cdots, a_{N-1,i}$. In this paper, the case where data is mapped onto pairs of subcarriers is considered, so $n = N/2$ and $a_{2M+1,i} = -a_{2M,i}$, where i represents the ith IFFT output data block. The demodulated subcarrier pairs will be weighted and added to obtain the estimates of the transmitted data sequence.

In its simplest form, PCC-OFDM is not bandwidth efficient. To overcome this drawback, we can use PCC-OFDM with symbols overlapped in the time domain (Overlap PCC-OFDM), so that higher bandwidth efficiency can be achieved [4][5]. The frequency offset estimation for Overlap PCC-OFDM has been investigated in a recent paper [6]. In this paper, we will not consider the case of Overlap PCC-OFDM in detail.

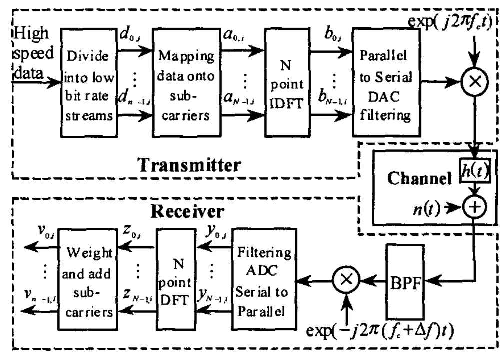

Fig.1. Block diagram of a PCC-OFDM system

This paper is organized as follows, in Section 2, the interchannel interference (ICI) and intersymbol interference (ISI) caused by timing and frequency offset are investigated. In Section 3 and 4, the time and frequency offset estimators are introduced. In Section 5, numerical simulation results are presented and the performance of the estimators is evaluated. In Section 6, the application of the estimators in normal OFDM systems is discussed. Conclusions are drawn in Section 7.

2. ICI, ISI and PCC-OFDM signals in the presence of time and frequency offset

Time and frequency offset will cause ICI and ISI in an OFDM system [7][8]. The ICI and ISI caused by time offset in PCC-OFDM systems have been investigated in [9]. It is shown, in PCC-OFDM the ICI and ISI caused by time and frequency offset have been substantially reduced. The significant terms of ICI and ISI in PCC-

OFDM cross fewer subcarriers than in OFDM. Therefore, PCC is less sensitive to time and frequency offsets. However, the estimation of these offsets is still crucial in PCC because of the remaining significant ICI and ISI terms.

Figure 2 illustrates the time offset to be considered in this paper. In PCC-OFDM the energy of symbol is concentrated in the middle of the symbol period because of the windowing effect due to mapping [4]. The time offset τ is caused by the misalignment of the FFT window and the received symbol, where $\tau = pT/N$, T is the symbol period and p is an integer.

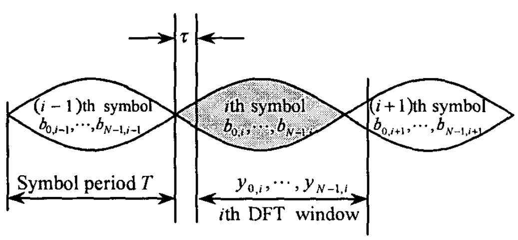

Fig.2. Timing offset caused by the misalignment of the FFT window and the received symbol

In the presence of a timing offset illustrated in Figure 2, the input of the ith FFT window contains components from two adjacent received symbols, e.g. from the ith and $(i+1)$th received symbols. Consequently, the ith FFT output will contain the data values from the adjacent data blocks to be transmitted. The mth output of the FFT is given by

$$z_{m,i} = \sum_{k=0}^{N-1} y_{k,i} \exp\left(\frac{-j2\pi mk}{N}\right) \tag{1}$$

where $y_{k,i}$ is the kth sample in the ith FFT window, which is defined by

$$y_{k,i} = \begin{cases} b_{k+p,i}, & 0 \le k \le N\text{-}1\text{-}p \\ b_{k-N+p,i+1}, & N\text{-}p \le k \le N\text{-}1 \end{cases} \tag{2}$$

The components from the ith and $(i+1)$th FFT output are given by

$$b_{k+p,i} = \sum_{l=0}^{N-1} a_{l,i} \exp\left(\frac{j2\pi(k+p)l}{N}\right) \tag{3}$$

$$b_{k-N+p,i+1} = \sum_{l=0}^{N-1} a_{l,i+1} \exp\left(\frac{j2\pi(k-N+p)l}{N}\right) \tag{4}$$

Using $a_{2M+1,i} = -a_{2M,i}$ and $v_{M,i} = (z_{2M,i} - z_{2M+1,i})/2$ for PCC-OFDM, then the output of the weighting and adding block due to one input in the ith received symbol is given by [9]

$$v_{M,L,i,i} = \frac{1}{2N} d_{M,i} \exp\left(\frac{j4\pi Lp}{N}\right) \left[\sum_{k=0}^{N-1-p} \exp\left(\frac{j4\pi k(L-M)}{N}\right)\left(1-\exp\left(\frac{j2\pi(k+p)}{N}\right)\right) \left(1-\exp\left(\frac{-j2\pi k}{N}\right)\right)\right] \tag{5}$$

Figure 3 shows the graph of $|v_{M,L,i,i}|^2$, the amplitude peaks at the point $\tau = 0$ and monotonically decreases with the timing offset. It is shown that all cross terms for $L \ne M$ are negligible, i.e. $|v_{M,i}|^2 \approx |v_{M,i,i}|^2 + |v_{M,i,i+1}|^2$. Therefore we may correct the timing offset by maximizing the amplitude of $|v_{M,i,i}|^2$.

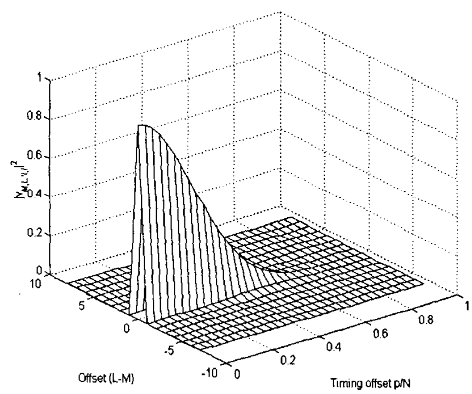

Fig.3. Interference power as a function of timing offset and subchannel difference

Note in the figure, the components from the $(i+1)$th received symbol have not been taken into account. The $(i+1)$th symbol affects the amplitude symmetrically at the point of $p/N = 0.5$. The expression of the ith output of the weighting and adding block, $v_{M,i}$, has components from both ith and $(i+1)$th symbols.

The ICI caused by frequency offset in PCC has been investigated in [9][10]. In OFDM, the ICI is significant in all subcarriers In contrast, the significant terms cross only very few subcarriers in PCC-OFDM [1].

3. Timing offset estimator

The timing offset is estimated by calculating a metric based on the frequency domain modulated data pairs. The timing metric is given by the function

$$F(\tau)=\frac{1}{4N}\sum_{M=0}^{N/2-1}\left|z_{2M,i}-z_{2M+1,i}\right|^2=\frac{1}{N}\sum_{M=0}^{N/2-1}\left|v_{M,i}\right|^2 \qquad (6)$$

where τ is the timing offset. $z_{2M,i}$ and $z_{2M+1,i}$ are a pair of demodulated subcarriers from the receiver FFT output in the $2M$th and $(2M+1)$th channel. The correct timing point is where the timing metric (6) is maximal. In the ideal case, when there is no timing offset or other distortions, $z_{2M,i}=-z_{2M+1,i}$ and the metric reaches a maximal value of unity. A timing offset interrupts this balance and causes the impairments of the demodulated subcarrier pairs.

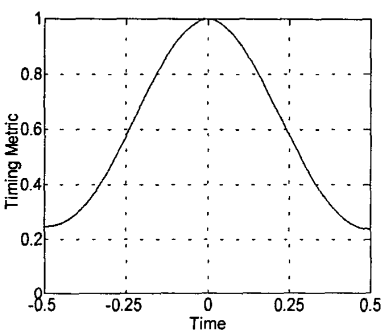

Fig. 4. Timing metric as a function of time

Figure 4 shows the timing metric as a function of the timing offset under the condition of no noise and distortion. The timing offset is shown as a fraction of the symbol period T. The timing metric reaches its maximal point when the timing error is zero. The maximal point (time 0 in the figure) is taken as the start of the useful part of a symbol.

4. Frequency offset estimator

The frequency offset estimation in PCC-OFDM is based on the impairments due to the frequency offset in the demodulated subcarrier pairs. In PCC-OFDM all subcarrier pairs can be used for the frequency offset estimation. By averaging all estimates over a symbol the variance of the estimator can be substantially reduced. The frequency offset is estimated by the function

$$F(\Delta fT)=\frac{1}{N/2}\sum_{M=0}^{N/2-1}\mathrm{Re}\left(\frac{z_{2M+1,i}+z_{2M,i}}{z_{2M+1,i}-z_{2M,i}}\right) \qquad (7)$$

where $\mathrm{Re}(\bullet)$ is the real part of a complex number. ΔfT represents the normalized frequency offset. $z_{2M,i}$ and $z_{2M+1,i}$ is a pair of demodulated subcarrier pair. When $|\Delta fT|\le 0.5$, the estimator is an approximately linear function of the frequency offset.

Figure 5 shows the frequency offset estimator as a function of the frequency offset for $N=1024$. The effect of timing offset on the performance of the frequency offset estimator will be reported in Section 5.

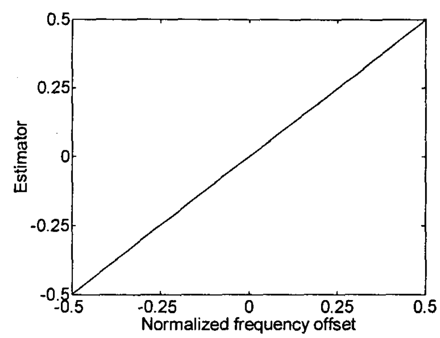

Fig.5. Frequency estimator as function of the frequency offset

It is shown in [10], the frequency offset in normal OFDM can also be estimated by using a pair of demodulated PCC coded subcarriers within a symbol. The advantage of using only one pair of subcarriers in this case is that it is bandwidth efficient because all other subcarriers can be used for carrying information data. However the variance of the estimator is large.

5. Simulation results

To evaluate the performance of the estimators for timing and frequency offsets, simulations were performed for different cases, timing estimation with frequency offset and noise; frequency offset estimation with timing errors and noise. In all cases, 4QAM modulation with 1024 subcarriers is used.

Figure 6 shows the time metric as a function of the time for different frequency offset. In the presence of a frequency offset, the maximal point of the time metric remains at $\tau = 0$, however the maximum value of the time metric is reduced. The larger the frequency offset, the larger the amplitude reduction. This is because a frequency offset also breaks the balance between the two subcarriers of a PCC subcarrier pair.

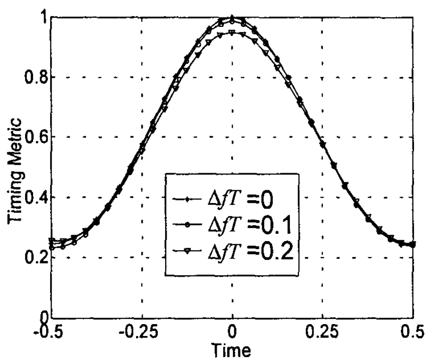

Fig.6. Variance of time metric at correct time as a function of frequency offset

Figure 7 shows the mean at the correct timing of the time offset estimator as a function of the channel additive white Gaussian noise (AWGN). The mean value increases as E_b/N_0 increases, when E_b/N_0 is low, there is an amplitude reduction up to 17% while the mean value tends to unity when E_b/N_0 is high. The reason for the amplitude reduction is that the channel noise causes the impairment of the demodulated subcarrier pairs.

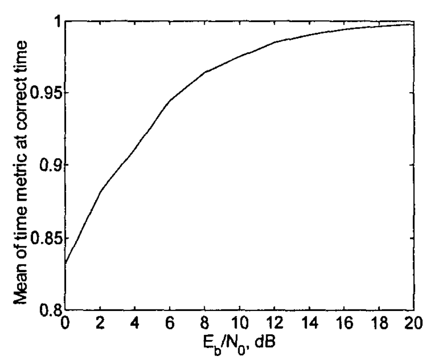

Fig.7. Mean value of time metric at correct time as a function of AWGN noise

Figure 8 shows the variance of the timing metric at the correct point as a function of channel noise. It is shown that the variance of the estimator is very small under condition of a high signal-to-noise ratio.

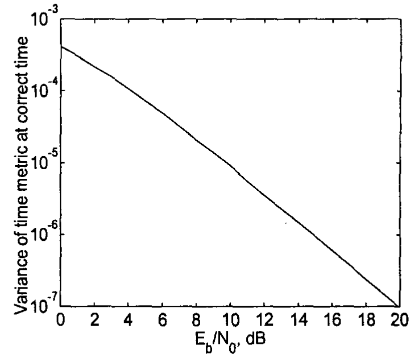

Fig.8. Variance of time metric at correct time as a function of AWGN

Figure 9 shows the variance of the frequency offset estimator as a function of the noise level. The variance of the estimator depends on the frequency offset. In the figure, three examples are presented for normalized frequency offset $\Delta fT = 0$, 0.2 and 0.4 respectively.

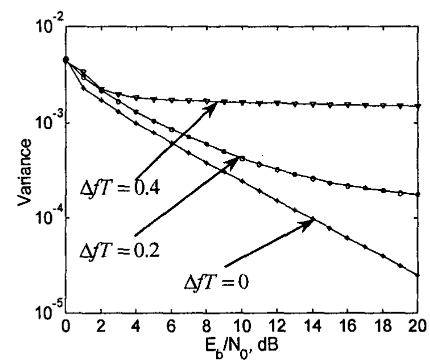

Fig.9. Variance of the frequency estimator as a function of E_b/N_0

Figure 10 shows the frequency offset estimator as a function of the frequency offset in the presence of different timing offset. When the normalized timing offset is less than a half symbol period, the linearity of the frequency offset estimator changes as the timing offset increases. However the estimator will still work. A timing offset of 0.25 will cause only negligible distortion on the frequency offset estimation.

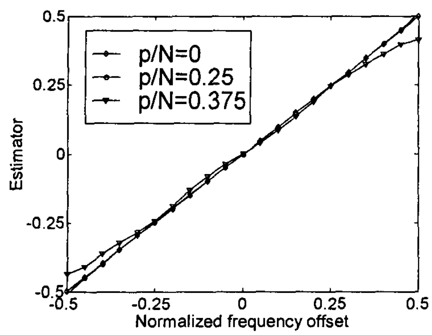

Fig.10. Frequency offset estimator as a function of frequency offset

6. Implementation in OFDM and Overlap PCC-OFDM

The estimators can also be used for the time and frequency offset estimation in normal OFDM systems. In this case, a training symbol must be transmitted for the estimation of the time and frequency offset. The data structure of the training symbol can be the same as a PCC-OFDM symbol or a symbol in which a number of subcarriers are mapped onto PCC subcarrier pairs. When only some of the subcarriers are used for pilot tones, the variance of the estimator will be larger than that if a whole symbol is being used. A comparison of the variance in the two cases has been given in [10].

When the PCC-OFDM symbols are overlapped in the time domain, the implementation is different from the PCC-OFDM case due to the existence of the overlapping components. The time metric works for Overlap PCC-OFDM, however, the estimation range is halved. The implementation of the frequency offset estimator in Overlap PCC-OFDM has been investigated in [5]. In this case, the frequency offset estimator works well but the variance of the estimator is larger.

7. Conclusion

Two estimators for timing and frequency offset for PCC-OFDM systems have been presented. The time and frequency offsets are estimated in the frequency domain using the demodulated subcarrier pairs in a PCC-OFDM symbol. The correct timing point is calculated by maximizing a timing metric. The frequency offset is estimated by calculating an estimator. No training sequence, pilot tones and cyclic prefix are required. The techniques can also be used in normal OFDM systems by inserting a PCC-OFDM pilot symbol in the OFDM signal. The frequency offset estimator can be used for Overlap PCC-OFDM, however, the linearity of the estimator will be slightly changed because of the overlapping components.

Reference

1 J. Armstrong, "Analysis of new and existing methods of reducing intercarrier interference due to carrier frequency offset in OFDM", *IEEE Trans. on Commun.,* March 1999, VOL 47, No.3, pp365-369.

2 K. A. Seaton and J. Armstrong, "Polynomial cancellation coding and finite differences", *IEEE transactions on Information Theory,* VOL. 46, No. 1, January 2000.

3 J. Armstrong, P. M. Grant, and G. Povey, "Polynomial Cancellation Coding of OFDM to reduce Intercarrier interference due to Doppler spread", *IEEE Globecom,* 8-12 November, 1998, pp. 2771-2776.

4 J. Armstrong, J. Shentu and C. Tellambura, "Frequency domain equalization for OFDM systems with mapping of data onto subcarrier pairs and overlapping symbol periods", *"Coding Communication and Broadcasting",* Edited by P. G. Farrell, Mike Darnell and Bahram Honary, 2000, Research Studies Press. pp217-228.

5 J. Armstrong and C. Tellambura, "Multicarrier CDMA systems using PCC-OFDM", *IEEE Vehicular Technology Conference,* May 2000, Tokyo.

6 J. Shentu and J. Armstrong, "Blind Frequency Offset Estimation for PCC-OFDM with Symbols Overlapped in the Time Domain". Accepted by IEEE ISCAS2001, May 6-9, Sydney.

7 Timothy M. Schmidl and Donald C. Cox, 'Robust Frequency and Timing Synchronization for OFDM', *IEEE Transactions on Communications,* 45(12): 1613-21, December 1997.

8 T. Pollet and M. Moeneclaey, "Synchronizability of OFDM Signals" *IEEE, Globecom'95.* May, 1995, pp2054-2058.

9 J. Armstrong, T. Gill and C. Tellambura, "Performance of PCC-OFDM with overlapping symbol periods in a multipath channel", IEEE *Globecom'* 00, pp87-91.

10 J. Shentu and J. Armstrong, "A New Frequency Offset Estimator for OFDM", *"Communication systems networks and digital signal processing",* edited by A. C. Boucouvalas, July 2000, UK. pp13-16.