# A New Frequency Offset Estimator for OFDM

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## ABSTRACT

In this paper we introduce a new method of carrier frequency offset estimation for Orthogonal Frequency Division Multiplexing (OFDM) communication systems. The method uses one or more pairs of adjacent subcarriers within an OFDM symbol. The pairs of subcarriers are modulated with equal amplitude and opposite polarity. In the absence of frequency offset, the received subcarrier pairs also have equal amplitude and opposite polarity. Frequency offset disrupts this balance in a predictable way. This is used as the basis for the new frequency offset estimator. When the normalized frequency offset is less than half of the subcarrier frequency spacing, the new estimator is shown to be an approximately linear function of carrier frequency offset. The method is particularly applicable to the technique of Polynomial Cancellation Coding (PCC) in which the data to be transmitted is mapped onto subcarrier pairs rather than individual subcarriers. A number of different implementations are possible depending on whether the basic modulation technique is ordinary OFDM, or PCC-OFDM. For use with ordinary OFDM pilot subcarrier pairs must be inserted. If the pilot pair is contained in a symbol that also carries data, when the frequency offset is not close to zero, inter-carrier interference (ICI) from the other subcarriers can result a large variance of the estimator. For PCC-OFDM the estimation can be based on the subcarrier pairs which are carrying data. The variance in this case is much reduced, both because of the reduced ICI of PCC-OFDM and because the estimate can be based on the average over a number of pairs. Simulation results are presented for a number of cases.

## 1. INTRODUCTION

Carrier frequency offset estimation plays an important role in OFDM communication systems because of their high sensitivity to carrier frequency offsets [1, 2]. Figure 1 shows the structure of an OFDM communication system.

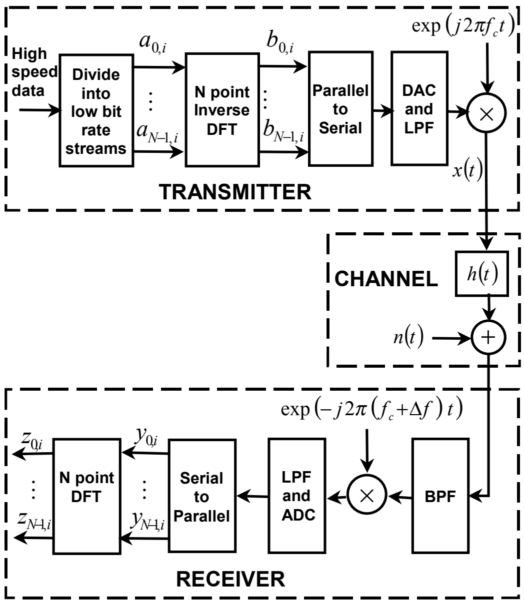


Fig.1. A block diagram of an OFDM communication system

The new carrier frequency offset estimator is based on the function:

$$F(\Delta fT) = \mathrm{Re}[(z_{m,i}+z_{m+1,i})/(z_{m,i}-z_{m+1,i})] \quad (1)$$

where $\mathrm{Re}[\bullet]$ is the real part of a complex value. $\Delta f$ represents the carrier frequency offset between the transmitter and receiver of an OFDM system, and $T$ is the symbol period. Then $\Delta fT$ represents the normalized carrier frequency offset. $z_{m,i}$ is the output of the receiver Discrete Fourier Transform (DFT) in the $m$-th channel in the $i$-th symbol period. The new carrier frequency offset estimation method uses two adjacent subcarriers in the OFDM signal. The two subcarriers must be modulated with same amplitude and opposite polarity. That is $a_{m,i} = -a_{m+1,i}$ where $a_{m,i}$ is $m$-th input to the transmitter Inverse Discrete Fourier Transform (IDFT) in *i*-th symbol period, and where the subcarriers with index *m* and $(m+1)$ are being used for frequency estimation. The method depends only on the adjacent pair of subcarriers having opposite polarity. It will work for any value of $a_{m,i}$ including complex values.

## 2. ANALYSIS

To analyze the performance of the new estimation method, the effect of carrier frequency offset on individual subcarriers must be calculated. In the ideal case, where an OFDM signal is transmitted without distortion, $z_{m,i}=a_{m,i}$, each output from the receiver DFT depends on the corresponding input to the transmitter IDFT. However, any carrier frequency offset will disrupt the orthogonality of the subcarriers and result in intercarrier interference (ICI). As a result each output depends on every input. For a normalized carrier frequency offset of $\Delta fT$ it can be shown that [2, 3]:

$$z_{m,i}=\frac{1}{N}\exp(j\theta_0)\times\sum_{l=0}^{N-1}a_{l,i}\frac{\sin(\pi(l-m+\Delta fT))}{\sin(\pi(l-m+\Delta fT)/N)}\times\exp(j\pi(N-1)(l-m+\Delta fT)/N) \quad (2)$$

where $\theta_0$ is the phase offset between the phase of the receiver local oscillator and the carrier phase at the start of the received symbol. $N$ is the number of subcarriers within one symbol. To simplify the analysis of the frequency synchronization algorithm, define a set of complex weighting coefficients, $c_{1-N}\cdots c_{N-1}$ where

$$c_{l-m}=\frac{\sin(\pi(l-m+\Delta fT))}{N\sin(\pi(l-m+\Delta fT)/N)}\times\exp(j\pi(N-1)(l-m+\Delta fT)/N) \quad (3)$$

Therefore $z_{m,i}$ can be represented as

$$z_{m,i}=\exp(j\theta_0)\sum_{l=0}^{N-1}c_{l-m}a_{l,i} \quad (4)$$

The implementation of the new estimation method will be considered for three different cases.

First, to demonstrate the performance of the estimator without any impairments caused by interference from other subcarriers, consider the very simple case where only two data inputs in the *i*-th symbol period are non-zero. That is $a_{m,i}=-a_{m+1,i}$. Because there are only two nonzero inputs, equation (4) can be rewritten as

$$z_{m,i}=\exp(j\theta_0)(c_0-c_1)a_{m,i} \quad (5)$$

Similarly, $z_{m+1,i}$ can be represented in terms of $c_{-1}$ and $c_0$. Substituting $z_{m,i}$ and $z_{m+1,i}$ in equation (1) gives

$$F(\Delta fT)=\mathrm{Re}[(c_{-1}-c_1)/(-c_1+2c_0-c_{-1})] \quad (6)$$

The estimator $F(\Delta fT)$ depends only on the complex weighting coefficients $c_{-1},c_0,c_1$, not on the transmitted data value $a_{m,i}$. Any pair of adjacent subcarriers would give the same result. Figure 2 shows the graph of $F(\Delta fT)$ as a function of normalized carrier frequency offset for $N=4$ and $N=16$. For $N\geq 16$, there is an approximately linear relationship between normalized frequency offset and the frequency offset estimator.

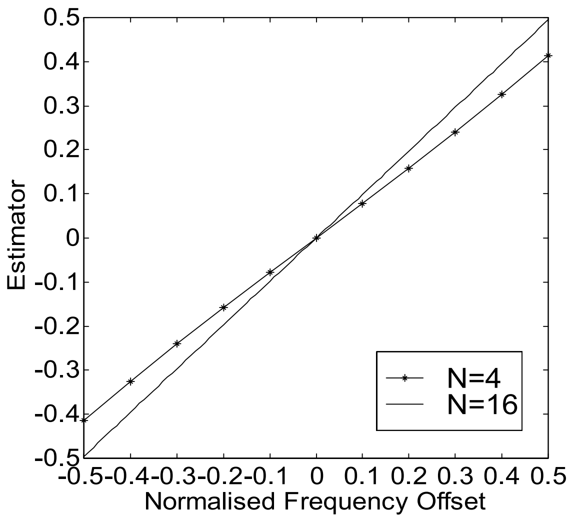


Fig.2. Estimator as a function of normalized frequency offset

This can be shown to be true for all large $N$, by using the approximations $(N-1)/N\approx 1$ and $\sin(\theta)\approx\theta$ for small $\theta$, combining (3) and (6) gives

$$F(\Delta fT)\approx\Delta fT \quad (7)$$

Figure 3 shows the variance of the estimator as a function of frequency offset for various cases. The variance was calculated by simulating 10,000 symbols, for $N=32$ and where 4-QAM was used to modulate the subcarriers. For this case the variance of the estimator is zero for all frequency offset.

A more practical case to consider, is where one pair of adjacent subcarriers is used for frequency estimation and the rest of the subcarriers are modulated with the random data that is to be transmitted. ICI from other subcarriers then results in jitter in the frequency estimate. The ICI does not affect the mean of the estimator, which still has the form shown in Figure 2. However, the variance is no longer zero. The variance is a function of carrier frequency offset. For zero frequency offset there is no ICI, the variance is zero.

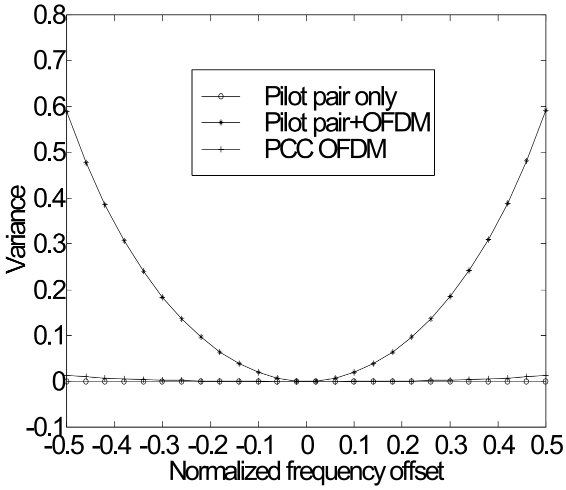


Fig. 3. Variance as a function of normalized carrier frequency offset carrier frequency offset

As the frequency offset increases, the ICI also increases with a resulting increase in the variance of the estimate.

A third important case is where is all of the subcarriers in a symbol are modulated in pairs with opposite polarities [4, 5, 6, 7], i.e. $a_{0,i} = -a_{1,i}$, $a_{2,i} = -a_{3,i}, \ldots,$ $a_{N-2,i} = -a_{N-1,i}$. This situation may occur where a special synchronization symbol is being transmitted or where PCC is being used to improve the properties of OFDM transmission [2, 3]. Frequency offset can be estimated using the average of a number of values of $F(\Delta fT)$ from different pairs of subcarriers within the same symbol period. Thus $F(\Delta fT)$ depends on the combination of transmitted random data values. Different combinations result in slight differences in $F(\Delta fT)$. Figure 3 shows that the variance of the estimator is only slightly greater than the first case, however, much more data is carried in the symbol.

## 3. PERFORMANCE WITH NOISE

The estimator was simulated for zero frequency offset, and varying amounts of Additive White Gaussian Noise (AWGN). It can be seen from (1) that when the values $z_{m,i}, z_{m+1,i}$ are very close to each other, the term $z_{m,i} - z_{m+1,i}$ will tend to zero, therefore the magnitude of the estimator becomes very large. As these large amplitude values are the result of noise rather than large frequency offset, an improved estimator was designed which limited the amplitude of the estimator to unity. The results for the three cases are shown in Figure 4. In the presence of noise, the performance of the estimator varies with different implementation cases. In Figure 4 the variance is plotted as a function of bit energy to noise ratio $E_b/N_0$. With PCC OFDM, the variance due to noise is much less than other cases.

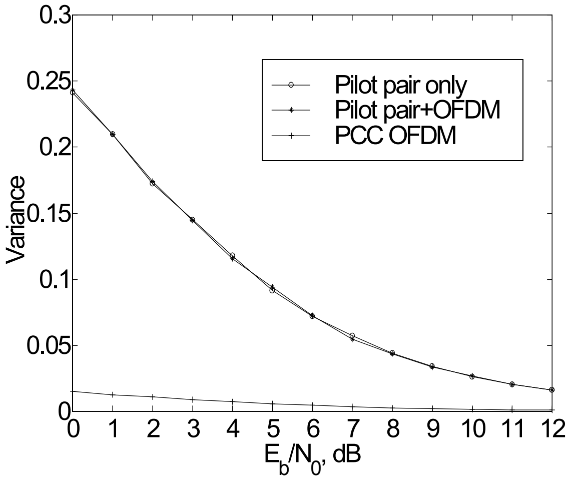


Fig. 4 Variance as a function of $E_b/N_0$

## 4. CONCLUSION

We have presented a new frequency offset estimator for OFDM systems. The estimate is made by applying a pair of pilot tones in an OFDM system, or by exploiting the symbol structure in a PCC-OFDM system. Three possible implementations have been analyzed. Simulation results show that the new estimator is effective in all three cases. In all cases the mean of the estimator is a linear function of frequency offset. The variance of the estimator depends on the effects of ICI and noise. Both of the components are much less for PCC-OFDM than for OFDM with a pair of pilot tones. The ICI is reduced because of the cancellation properties of PCC; the noise component is reduced because each estimate is based on the average over all of the subcarriers in a symbol rather than on one subcarrier pair.

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